



1. A curve  $C$  has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(a) Find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ . (5)

(b) Find an equation of the tangent to  $C$  at the point  $(3, -2)$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers. (2)

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3.

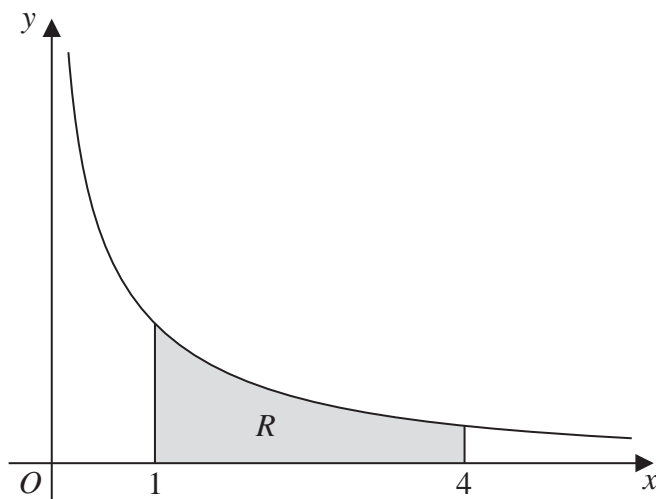


Figure 1

Figure 1 shows a sketch of part of the curve with equation  $y = \frac{10}{2x + 5\sqrt{x}}$ ,  $x > 0$

The finite region  $R$ , shown shaded in Figure 1, is bounded by the curve, the  $x$ -axis, and the lines with equations  $x = 1$  and  $x = 4$

The table below shows corresponding values of  $x$  and  $y$  for  $y = \frac{10}{2x + 5\sqrt{x}}$

$x$	1	2	3	4
$y$	1.42857	0.90326		0.55556

- (a) Complete the table above by giving the missing value of  $y$  to 5 decimal places. (1)
- (b) Use the trapezium rule, with all the values of  $y$  in the completed table, to find an estimate for the area of  $R$ , giving your answer to 4 decimal places. (3)
- (c) By reference to the curve in Figure 1, state, giving a reason, whether your estimate in part (b) is an overestimate or an underestimate for the area of  $R$ . (1)
- (d) Use the substitution  $u = \sqrt{x}$ , or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x + 5\sqrt{x}} dx$$
(6)

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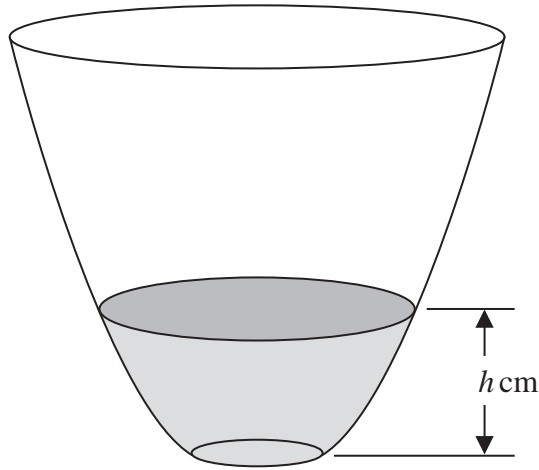








4.



**Figure 2**

A vase with a circular cross-section is shown in Figure 2. Water is flowing into the vase.

When the depth of the water is  $h \text{ cm}$ , the volume of water  $V \text{ cm}^3$  is given by

$$V = 4\pi h(h + 4), \quad 0 \leq h \leq 25$$

Water flows into the vase at a constant rate of  $80\pi \text{ cm}^3\text{s}^{-1}$

Find the rate of change of the depth of the water, in  $\text{cm s}^{-1}$ , when  $h = 6$

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**Question 4 continued**

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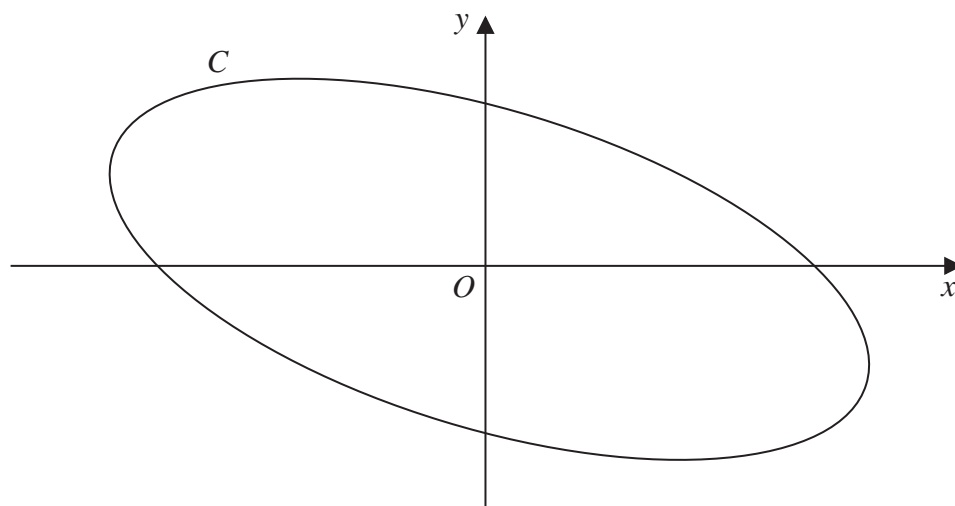
**Q4**

**(Total 5 marks)**



P 4 3 1 6 5 A 0 1 3 2 8

5.



**Figure 3**

Figure 3 shows a sketch of the curve  $C$  with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t, \quad 0 \leq t < 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3} \cos t \tag{3}$$

(b) Show that a cartesian equation of  $C$  is

$$(x + y)^2 + ay^2 = b$$

where  $a$  and  $b$  are integers to be determined.

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**Question 6 continued**

Lined area for writing the answer to Question 6.

**Q6**

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**(Total 12 marks)**



P 4 3 1 6 5 A 0 1 9 2 8

7.

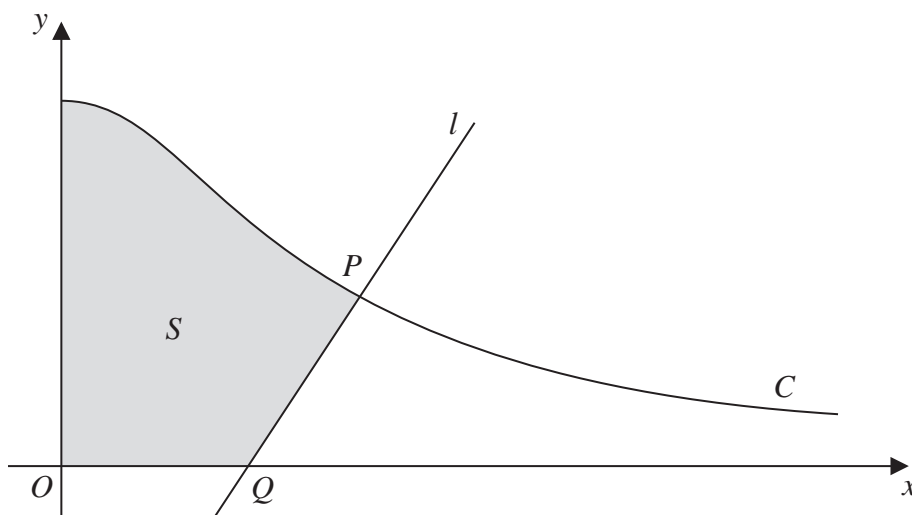


Figure 4

Figure 4 shows a sketch of part of the curve  $C$  with parametric equations

$$x = 3 \tan \theta, \quad y = 4 \cos^2 \theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The point  $P$  lies on  $C$  and has coordinates  $(3, 2)$ .

The line  $l$  is the normal to  $C$  at  $P$ . The normal cuts the  $x$ -axis at the point  $Q$ .

- (a) Find the  $x$  coordinate of the point  $Q$ . (6)

The finite region  $S$ , shown shaded in Figure 4, is bounded by the curve  $C$ , the  $x$ -axis, the  $y$ -axis and the line  $l$ . This shaded region is rotated  $2\pi$  radians about the  $x$ -axis to form a solid of revolution.

- (b) Find the exact value of the volume of the solid of revolution, giving your answer in the form  $p\pi + q\pi^2$ , where  $p$  and  $q$  are rational numbers to be determined.

[You may use the formula  $V = \frac{1}{3}\pi r^2 h$  for the volume of a cone.] (9)

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8. Relative to a fixed origin  $O$ , the point  $A$  has position vector  $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$

and the point  $B$  has position vector  $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$

The line  $l_1$  passes through the points  $A$  and  $B$ .

(a) Find the vector  $\overrightarrow{AB}$ . (2)

(b) Hence find a vector equation for the line  $l_1$  (1)

The point  $P$  has position vector  $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$

Given that angle  $PBA$  is  $\theta$ ,

(c) show that  $\cos \theta = \frac{1}{3}$  (3)

The line  $l_2$  passes through the point  $P$  and is parallel to the line  $l_1$

(d) Find a vector equation for the line  $l_2$  (2)

The points  $C$  and  $D$  both lie on the line  $l_2$

Given that  $AB = PC = DP$  and the  $x$  coordinate of  $C$  is positive,

(e) find the coordinates of  $C$  and the coordinates of  $D$ . (3)

(f) find the exact area of the trapezium  $ABCD$ , giving your answer as a simplified surd. (4)

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**Question 8 continued**

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